
Statistics in Morse theory of smooth functions

Vladimir I. Arnold

Steklov Mathematical Institute, Moscow, Russia

e-mail: arnold@mi.ras.ru

There are exactly 17746 topologically distinct Morse functions with 4 saddles on the two dimensional sphere.

This result basing on combinatorics of random graphs was obtained only two years ago within an investigation of Hilbert's 16th problem on topological classification of polynomials in real algebraic geometry.

Last year an American mathematician L. Nikolaescu proved the conjecture of V. Arnold claiming that the number of such functions with T saddles on the two-sphere grows with T as T in the power $2T$.

The methods of the proof go up to quantum field theory and mirror symmetry in physics.

Besides the description of these results the talk contains the counterparts in the theory of smooth functions on other manifolds, for example for functions on the torus and for trigonometric polynomials with prescribed Newton's diagram of a given affine Coxeter group.