On the Control of Liouville Equations

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Some important limitations standing in the way of the wider use of optimal control arise from the difficulty involved in properly weighting the cost of the apparatus required to implement the control in feedback form and from the fact that a control policy will judged on its performance relative to a distribution of initial states, rather than its performance starting from a single initial condition. With this in mind, we explore a class of optimization problems involving controlled Liouville equations. We argue in favor of replacing the usual control model $\dot{x} = f(x,u)$ by the related first order partial differential equation

$$\frac{\partial \rho(t,x)}{\partial t} = -\left\langle \frac{\partial}{\partial x}, f(x,u)\rho(t,x) \right\rangle$$

and for the consideration of performance measures which include *non trajectory dependent terms* such as the second and third terms on the right-hand side of

$$\eta = \int_0^T \int_S \rho(t,x)L(x,u)\, dx\, dt + \int_S \left(\frac{\partial u}{\partial x}\right)^2 dx + \int_0^T \left(\frac{\partial u}{\partial t}\right)^2 dt.$$

The talk will describe results on control and optimization in this context.