An Inactivation Principle in Biomechanics

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We consider the problem of analyzing which control procedure is performed by human brain during **pointing movements** of the arm. "Pointing movements" are movements in short time, that drive the end of the finger from certain initial position to certain terminal position, starting and ending with zero velocity.

Records from practical experiments show the following rather surprising behaviour: a bit after the middle of the duration T of the motion, one can see certain intervals of time where the agonistic and antagonistic muscles are simultaneously inactivated.

Another (minor) point is that the velocity profiles are not symmetric within the interval of time. In particular, maximum velocity is always reached between $0.44T$ and $0.49T$ (for upward movement).

The purpose of this lecture is to present a **general theory explaining these phenomena**. Mostly, the ingredients of the theory are Transversality Theory together with Pontriaguin's Maximum Principle (and also the Clarke's nonsmooth version of the maximum principle).

We consider mechanical systems with generalized coordinates $x \in \mathbb{R}^n$ and Lagrangian

$$L(x, \dot{x}) = \frac{1}{2}\dot{x}^T M(x)\dot{x} - V(x),$$

The equations of motion are given by substituting into Lagrange's equation,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = u + N(x, \dot{x}) = u + N(x, \dot{x}),$$

in which $u \in \mathbb{R}^n$ represents the vector of external generalized forces acting on the system as controls, and $N(x, \dot{x})$ are other exterior forces (including frictions for instance).

Hence we get a dynamics of the form $\ddot{x} = \phi(x, \dot{x}, u)$, where:

$$\phi(x, \dot{x}, u) = M(x)^{-1}(N(x, \dot{x}) - \nabla V(x) - C(x, \dot{x})\dot{x} + u), \tag{1}$$

where the Coriolis matrix $C(x,\dot{x}) \in M_n(\mathbb{R})$ is defined as

$$C_{ij}(x,\dot{x}) = \frac{1}{2}\sum_{k=1}^{n}\Big(\frac{\partial M_{ij}}{\partial x_k} + \frac{\partial M_{ik}}{\partial x_j} - \frac{\partial M_{kj}}{\partial x_i}\Big)\dot{x}_k.$$

For a control force or torque u and a motion $x(t)$, $t \in [0,T]$, the algebraic work of external forces is $W{=}\int udx = \int_0^T u\dot{x}dt$. The "practical" work of external forces (i.e. the energy spent via the external forces) is in fact $\int_0^T |u\dot{x}|dt$. The **absolute work** Aw of external forces is the sum (for all muscles) of such contributions.

In practice, the control generalized forces appear under the guise of agonistic-antagonistic actions, i.e. $u_i = v_i - w_i$, with $v_i, w_i \geq 0$. Moreover we assume certain dynamics on the agonistic and antagonistic actions. Finally, the absolute work is:

$$Aw = \sum_{i=1}^{n}\Big(\int_0^T |v_i\dot{x}_i|dt + \int_0^T |w_i\dot{x}_i|dt\Big).$$

Our theory is twofold:

1. With transversality arguments, we show that the criterion minimized (if any) cannot be smooth at $u = 0$ for inactivations appear.

In other terms, the presence of inactivations in practice implies the minimization of a term **like the absolute work.**

2. With the Maximum Principle, assuming the minimization of a criterion which is a **compromise between the absolute work and some other term (comfort term),** we prove that inactivations must appear.

Moreover, we show **the very strong fact that simultaneous inactivation** of both agonistic and antagonistic muscles must appear.

Also, certain classical phenomena from biomechanics, such as the "triphasic pattern" are obtained, as by-products of the theory.