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## Focal Stability

Charles Pugh

*Department of Mathematics, University of Toronto, Toronto, Canada*

e-mail: `cpugh@math.utoronto.ca`

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Generically, how many geodesic loops of the same length originate at a common point? Ivan Kupka, Mauricio Peixoto, and I answer this and related questions. We give a fairly sharp bound. Abraham's Bumpy Metric Theorem and its proof by Anosov play a role, as does our focal stability conjecture.

This last is a kind of structural stability conjecture for the focal decomposition of the generic exponential map. Namely, we consider a tangent space to a Riemann manifold and divide it into the subsets of vectors having exactly  $k$  other vectors of equal length and equal exponential image. These subsets are the "focal components", and they give "the focal decomposition" of the tangent space. We conjecture that generically, the focal decomposition is topologically stable under perturbations of the Riemann structure. In some cases we have verified the conjecture. Bounding the number of geodesic loops of equal length is a first step toward proving the conjecture in general.

## References

- [1] I. Kupka, M. Peixoto, and C. Pugh, "Focal Stability of Riemann metrics" *J. reine angew. Math.*, 593, 31-72, (2006).