Singularly perturbed problems with boundary and internal layers

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Nonlinear singularly perturbed PDE's which have solutions with boundary and internal layers are of increasing interest because of many applications of practical importance. Our talk is mostly devoted to nonlinear singularly perturbed parabolic equations. Physically, these problems may be interpreted as models for reaction-diffusion and reaction-diffusion-advection systems in chemical kinetics, synergetics, astrophysics, biology, et. al. The solutions of these problems often feature a narrow boundary layer region of rapid change as well as internal layers of different types (contrast structures), and it needs to develop new asymptotic methods to investigate them formally as well as rigorously.

We present our extension of the well-known boundary layer functions method to construct the asymptotics of solutions of different classes of problems with internal layers. In order to illustrate our ideas we consider some classes of singularly perturbed problems, which include the problems of generation of sharp internal layers, their propagation and formation of stable stationary or periodic internal layers. These results is father development of our investigations of contrast structures which were published in the review paper [1].

Another class of singularly perturbed problems which will be discussed in our talk, are the problems in the case of intersection of the roots of the degenerate equation (this case is also called the case of exchange of stabilities). Previous results in this direction are represented in the review paper [2]. Here we will describe new results concerning solutions of this class of problems, their asymptotics, stability and formation.

Our rigorous investigation of the considered problems is based on asymptotic method of differential inequalities. The basic idea of this approach is to construct lower and upper solutions to the value problem by using

formal asymptotics. For all considered problems we state the existence of the solutions, estimate the accuracy of the asymptotics and investigate their stability and process of formation.

References

[1] A.B. Vasilieva, V.F. Butuzov, N.N. Nefedov, "Contrast structures in singularly perturbed problems" (in russian), *Fundamentalnaja i prikladnala matematika*, 4, No. 3, 799–851 (1998).

[2] V.F. Butuzov, N.N. Nefedov, K.R. Schneider, "Singularly Perturbed Problems in Case of Exchange of Stabilities", *Journal of Mathematical Sciences*, 121, No. 1, 1973–2079 (2004).