
Legendre singularities and implicit ODE

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The classification of singularities of first order implicit differential equations in one variable due to A.Poincare, M.Chibrario, V.Arnold, J.W.Bruce, A.Davydov, and others is a famous application of singularity theory. It forms now a separate chapter in the theory of ordinary differential equations. A version of the notion of an implicit equation called web is of interest in physics and geometry. Many authors were interested in particular cases of implicit differential equation related to Hamiltonian mechanics and physics.

The theory of implicit differential equations is based on Poincare's geometrical construction of projections of surfaces embedded into contact three-space.

Much less is known on implicit systems of ODE. In a recent series of papers A.Remizov described some basic properties of Poincare's construction in odd-dimensional space corresponding to an implicit system of simultaneous differential equations.

In this talk we discuss some multi-dimensional counterpart of the classical theory, namely we classify low dimensional generic singularities of first integrals for implicit systems of ordinary differential equations and of slow-fast dynamical systems.

It happens that the principle class of the singularities described below consists of Legendre projections of singular Legendre varieties called open Whitney umbrellas which arise in many other geometrical and physical applications of the singularity theory. They were studied by A.Givental, G.Ishikawa, and V.Zakalyukin.